1

(a) $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

Plug into $y'' - 2y' - 3y = 9x^2 + 1$ to get

$$2A - 2(2Ax + B) - 3(Ax^2 + Bx + C) = 9x^2 + 1$$

$$2A - 4Ax - 2B - 3Ax^2 - 3Bx - 3C = 9x^2 + 1$$

$$\underbrace{-3A}_{-3A=9}x^2 + \underbrace{(-4A - 3B)}_{-4A-3B=0}x + \underbrace{(2A - 2B - 3C)}_{2A-2B-3C=1} = 9x^2 + 1$$

$-3A = 9$

$-4A - 3B = 0$

$2A - 2B - 3C = 1$

$\rightarrow$ $A = -3$

$\rightarrow$ $-4(-3) - 3B = 0$

$-3B = -12$

$B = 4$

$\rightarrow$ $2(-3) - 2(4) - 3C = 1$

$-6 - 8 - 3C = 1$

$-3C = 15$

$C = -5$

$y_p = -3x^2 + 4x - 5$

(b) $y = y_h + y_p = c_1 e^{3x} + c_2 e^{-x} + (-3x^2 + 4x - 5)$

(2) We did this problem in class on 3/24. See the notes in the calendar on the webpage.

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(3) HW 10 - #1(b)

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(4)

$f(x) = x^4 \leftarrow f(2) = 2^4 = 16$

$f'(x) = 4x^3 \leftarrow f'(2) = 4(2)^3 = 32$

$f''(x) = 12x^2 \leftarrow f''(2) = 12(2)^2 = 48$

$f'''(x) = 24x \leftarrow f'''(2) = 24(2) = 48$

$f''''(x) = 24 \leftarrow f''''(2) = 24$

$f'''''(x) = 0 \leftarrow$ zero from this point on

$$x^4 = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3 + \frac{f''''(2)}{4!}(x-2)^4$$

$$x^4 = 16 + 32(x-2) + \frac{48}{2}(x-2)^2 + \frac{48}{6}(x-2)^3 + \frac{24}{24}(x-2)^4$$

$$x^4 = 16 + 32(x-2) + 24(x-2)^2 + 8(x-2)^3 + (x-2)^4$$

radius of convergence $r = \infty$ since its a polynomial

5

We have

$$\boxed{x_0 = 0}$$

$$\boxed{\begin{aligned} &y'' - xy' + y = 0 \\ &y'(0) = 1 \\ &y(0) = 1 \end{aligned}}$$

The coefficients are

$-x = 1 - x + 0x^2 + 0x^3 + \dots$

$1 = 1 + 0x + 0x^2 + 0x^3 + \dots$

$0 = 0 + 0x + 0x^2 + 0x^3 + \dots$

These are polynomials so they have radius of convergence $r = \infty$.

So we get a power series solution

$$y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \frac{y''''(0)}{4!}x^4 + \dots$$

with radius of convergence $r = \infty$.

Have

$$\boxed{\begin{aligned} &y(0) = 1 \\ &y'(0) = 1 \end{aligned}}$$

given

$$\boxed{\begin{aligned} &y'' = xy' - y \\ &y''(0) = 0 \cdot (y'(0)) - (y(0)) = 0 - 1 = -1 \end{aligned}}$$

$y''(0) = -1$

$$\boxed{\begin{aligned} y''' &= 1\cdot y' + xy'' - y' = xy'' \\ y'''(0) &= 0\cdot(y''(0)) = 0 \end{aligned}}$$

$\leftarrow y'''(0) = 0$

$$\boxed{\begin{aligned} y'''' &= 1\cdot y'' + xy''' \\ y''''(0) &= 1\cdot(y''(0)) + 0\cdot(y'''(0)) \\ &= 1\cdot(-1) + 0 \\ &= -1 \end{aligned}}$$

$\leftarrow y''''(0) = -1$

(a)

$$y(x) = 1 + 1\cdot x + \frac{(-1)}{2!}x^2 + \frac{0}{3!}x^3 + \frac{-1}{4!}x^4 + \cdots$$

$$y(x) = 1 + x - \frac{1}{2}x^2 - \frac{1}{24}x^4 + \cdots$$

(b) $r = \infty$